ALGEBRAIC THINKING SKILL OF HIGH SCHOOL STUDENTS WITH DISCOVERY LEARNING MODEL

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Abstract. *This study aims to analyze the algebraic thinking skill of high school students with discovery learning model. The indicators of algebraic thinking skill are generalization, abstraction, dynamic thinking, analytic thinking, organizing, and modeling. Class X-2 at SMA Plus Ulumul Quran Al Mustofa was purposively chosen as the subject of the study, where the students had to answer the algebraic thinking skill test. Based on the test results, 1 student who got the highest score, 1 student who got a score around the average, and 1 student who got the lowest score were selected. The data were obtained from the results of tests and interviews. After the learning process with discovery learning model, students with the highest score are able to implement the generalization, abstraction, dynamic thinking, organizing, and modeling indicators in algebraic thinking skill. Students with scores around the average score are able to implement generalization and dynamic thinking indicators in algebraic thinking skill. The student with the lowest score was only able to implement the generalization indicator in algebraic thinking skill. The learning process influences students' algebraic thinking skills, so teachers are advised to be able to apply the right learning model to train and develop students' algebraic thinking skills.*

Keywords: *Algebraic Thinking; Discovery Learning; Generalization*

1. INTRODUCTION

Human daily activities cannot be separated from thinking activities. According to Gilmer in (Susanti, 2018), thinking is a problem solving. Thinking is also the process to replace an activity that appears physically using ideas or symbols. Hudojo (1990) stated that when students learn mathematics, students do thinking process, so that mathematics cannot be separated from thinking process. Cahyaningtyas, Novita, & Toto (2018) emphasized that the form of thinking that students often do in learning mathematics is algebraic thinking.

Driscoll (1999) defines algebraic thinking by stating "algebraic thinking could be thought of as the capacity to represent quantitative situations so that relations among variables become apparent". According to Kieran (2004), "algebraic thinking can be interpreted as an approach to quantitative situations that emphasizes the general relational aspects with tools that are not necessarily lettersymbolic, but which can ultimately be used as cognitive support for introducing and for sustaining the more traditional discourse of school algebra". While Lew (2004) stated that "success in algebra depends on at least six kinds of mathematical thinking abilities as follows: generalization, abstraction, analytic thinking, dynamic thinking, modeling, and organization". Lew stated that algebraic thinking consists of several thinking activities, namely generalization, abstraction, analytic thinking, dynamic thinking, modeling, and organization.

The algebraic thinking process is based on basic mathematical ideas and concepts that are used to solve problems. Algebraic thinking skills are important for students

because they help them to develop and generalize solutions of a problem. This is in line with the objectives of learning mathematics in Kurikulum Merdeka, that is understanding mathematical understanding and having procedural skills, using mathematical reasoning and proof, improving mathematical problem-solving skills, developing mathematical communication and representation skills, improving mathematical connection skills, and having an attitude of appreciating the usefulness of mathematics in life.

Students' algebraic thinking skill cannot be separated from the learning process. The learning model that is applied in the learning process is expected to be able to optimize students' understanding, so that students have good thinking skills. The characteristics of an appropriate learning model are those that can direct students to actively discover the concepts they are learning. According to Thorset (2002), the principle of discovery learning is not to provide knowledge directly to students, but students must discover new knowledge by themselves. Hosnan (2014) states that discovery learning is a model for developing active student learning methods, where students discover and investigate by themselves, so that the results obtained will last a long time in memory or are not easily forgotten by students. The discovery learning model is expected to provide students with the opportunity to discover lesson concepts independently, so that they can optimize their understanding.

Researchers have studied students' algebraic thinking skills, including Susanti (2018) who stated that as many as 76 out of 109 grade XI students experienced difficulties and misconceptions in algebra on the topic of sequences. Cahyaningtyas, Novita, & Toto (2018) revealed that students often have difficulty in predicting patterns and chunking information, students are also less able to make mathematical modeling, use mathematical symbols and algebraic operations. Sari, Fuad, & Ekawati (2020) in their research stated that there are differences in algebraic thinking skills between high, medium, and low-skill students based on indicators of generalization algebraic thinking, abstraction, analytic thinking, dynamic thinking, modeling, and organizing. It can be seen that there are still difficulties faced by students regarding the algebraic thinking process. Therefore, research related to students' algebraic thinking skills still needs to be done, especially when students learn with a particular learning model. Thus, this study aims to analyze the algebraic thinking skills of high school students by applying the discovery learning model.

2. RESEARCH METHODS

2.1 Type of Research

This is study descriptive with approach qualitative. The purpose of this study is to analyze algebraic thinking skill of high school students on the material logarithm with the discovery learning model.

2.2 Subject Study

This study used purposive sampling technique to determination subject. The subjects who involved in this study is 17 students in class X-2 at SMA Plus Ulumul Quran Al Mustofa. The subjects chosen in study are 3 students with details of 1 student gets the highest score, 1 student gets score around the average, and 1 student gets the lowest score.

2.3 Data Collection Technique

Data collected in this study are qualitative data that obtained through results test and interview. There are 6 questions related with material logarithm in the test. The questions are customized with indicators of algebraic thinking proposed by Lew (2004). Table 1 presents the question grid of the algebraic thinking test.

Table 1. Question Grid of Algebraic Thinking Test

Algebraic Thinking	Question Indicator	Question Number
Generalization	Students are able to recognize general patterns of logarithmic properties and apply them to cases with more than two variables.	1
Abstraction	Students understand the general relationship between logarithms and exponents, isolate them from numerical examples and see general patterns.	2
Dynamic Thinking	Students are able to see how changing the base affects the value of the logarithm and realize the flexibility of the concept of logarithms when the base changes.	3
Analytic Thinking	Students are able to analyze common errors in solving logarithmic equations and reconstruct the problem-solving process correctly.	4
Organizing	Students are able to organize the right method in solving logarithmic equations and plan an efficient sequence of steps.	5
Modeling	Students are able to apply logarithmic forms in the context of sales result modeling.	6

Researchers also use guidelines interview to get more information related to algebraic thinking skill of students. Before used, all of the instruments in this study validated by expert validators.

2.4 Data Analysis Technique

Analysis in this research done through data categorization, data reduction, data presentation, data interpretation, and extraction conclusion. The results of data analysis in this study is description related algrebraic thinking skill of students with the discovery learning model.

3. RESULTS AND DISCUSSION

Activity learning with discovery learning models and approaches scientific on the material logarithm conducted in class X-2 SMA Plus Ulumul Quran Al Mustofa. The learning activity is accordance with stages of discovery learning model, started with stimulus that requires student to think of solutions for a problem. The next step is identification problem, then collect the relevant data to be processed until formed generalization. After that, the generalization results being verified or proven, and at the final stage, student will get the conclusion (Dahlan, Priatna, & Nurlaelah, 2023). Observing, asking, collecting information, associating, and communicating which are parts of approach scientific (Dahlan, Priatna, & Nurlaelah, 2023) carried out in learning activities.

After passed all of the stages of discovery learning, the students answer the algebraic thinking test. Based on the results of the test, 3 students were selected as subject study. Table 2 presents their results. Based on the scores in Table 2, S1 gets the highest score, S2 gets score around the average, and S3 gets the lowest score.

Table 2. Results of Algebraic Thinking Test

Student	Algebraic Thinking Indicator						Total
	Generalization	Abstraction	Dynamic Thinking	Analytic Thinking	Organizing	Modeling	
S1	10	20	20	10	15	0	75
S2	10	0	20	0	0	0	30
S3	10	0	0	0	0	0	10

3.1 Algebraic Thinking Skill of Student with the Highest Score

It can be seen from Figure 1 that S1 understands the information that provided in question number 1 based on writing what is known and asked. This is in accordance as stated by Wilujeng (2017) that student understand problem in question with write what is known and asked. Then from the answer written by S1, was obtained that S1 is capable generalize characteristic logarithm for three variables based on characteristic the logarithm presented in the question. The steps taken by S1 are in line with the results presented by Yusrina & Masriyah (2019) in their research, where students use existing patterns, that is characteristic logarithm with two variables, for determine characteristic logarithm with 3 variables. So, the indicator generalization in algebraic thinking skill was satisfied by S1.

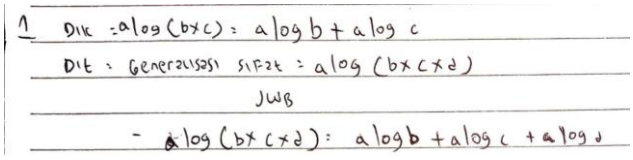

Figure 1. S1's Answer to Question Number 1

It is obtained on Figure 2 that S1 is capable simplify expression logarithms presented in the questions number 2, namely $\log_a a^5$. Then S1 is also able to abstract the properties of logarithms for more complex cases in general, that is $\log_a a^n$. So, the indicator abstraction in algebraic thinking skill was satisfied by S1.

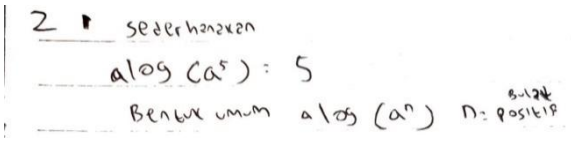

Figure 2. S1's Answer to Question Number 2

It can be seen in Figure 3 that S1 understands the information that provided in question number 3 based on writing what is known and asked. S1 is able to determine the numerus for expression logarithm $\log_2 x = 3$ correctly. Then, S1 can also determine the numerus for expression logarithm when the basis changed becomes 4. So, indicator dynamic thinking in algebraic thinking skill was satisfied by S1.

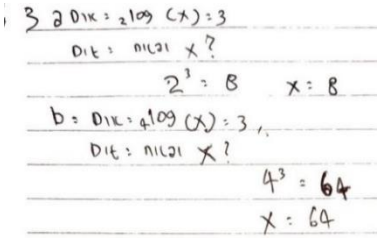

Figure 3. S1's Answer to Question Number 3

It is obtained from Figure 4 that S1 understands the information that provided in question number 4 based on writing what is known and asked. However, S1 has not capable fulfil indicator analytic thinking because S1 only conclude one solution for equality the logarithm presented, that is 5, whereas there are two solutions for equality, that is 5 and -5. When being interviewed, S1 admitted that he did not realize the solution in negative number.

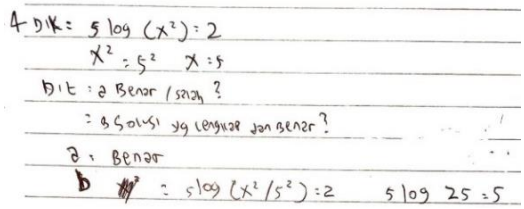

Figure 4. S1's Answer to Question Number 4

It can be seen in Figure 5 that S1 understands the information that provided in question number 5 based on writing what is known and asked. S1 is also able to determine the solution of x, but S1 did not write the stages of organizing in determine solution of x. Nurjanah & Hakim (2020) state that students who get answer not necessary means that they satisfy the mathematically skill. Therefore, S1 was interviewed to to get more information related to algebraic thinking skill of S1. Here are the interview results with S1.

Q: What methode did you use to get the solution $x = 62$*?*

S1: On the question known that the exponent is 3 and the base is 4, so $4^3 = 64$*. If* $x + 2 = 64$*, then* $x = 62$*.*

Based on the interview results, it can be known that S1 can explain the steps he took in finish equality logarithms. So, the indicator organizing in algebraic thinking skill was satisfied by S1.

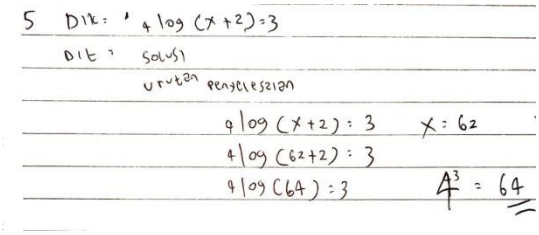

Figure 5. S1's Answer to Question Number 5

On the question number 6, the answer is still wrong. But, S1 was interviewed to get more information related to algebraic thinking skill of S1. Here are the interview results with S1.

Q: Is there any difficulty in answer the question number 6?

S1: While working question number 6, the time of the test was almost finished. So, I just do what I can

Based on the interview results, it can be known that S1 did not answer question number 6 correctly because the time of the test was almost finished. Then, S1 was asked to answer question number 6 and obtained we get the answer in Figure 6. From the answer in Figure 6, it is known that S1 is capable finish problem use function logarithm correctly, so indicator modeling in algebraic thinking skill was satisfied by S1.

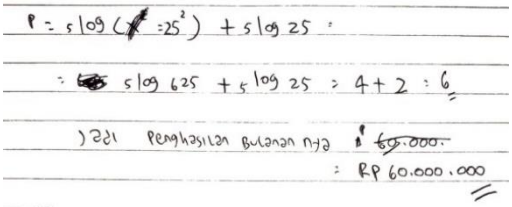

Figure 6. S1's Answer to Question Number 6

In general, S1 satisfy the indicator generalization, abstraction, dynamic thinking, organizing, and modeling in algebraic thinking skill. From the results, it is known that S1's algebraic thinking skill is more complete than results research conducted by

Warsitasari (2015), where students just satisfy indicator generalization, abstraction, thinking dynamic, and modeling in algebraic thinking skill.

3.2 Algebraic Thinking Skill of Student with Score Around the Average

It can be seen in Figure 7 that S2 understands the information that provided in question number 3 based on writing what is known and asked. This is in accordance as stated by Wilujeng (2017) that student understand problem in question with write what is known and asked. Then from the answer written by S2, was obtained that S2 is capable generalize characteristic logarithm for three variables based on characteristic the logarithm presented in the question. The steps taken by S2 are in line with the results presented by Yusrina & Masriyah (2019) in their research, where students use existing patterns, that is characteristic logarithm with two variables, for determine characteristic logarithm with 3 variables. So, the indicator generalization in algebraic thinking skill was satisfied by S2.

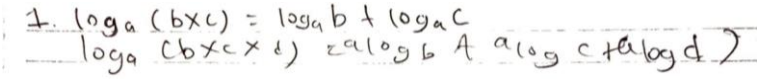

Figure 7. S2's Answer to Question Number 1

It is obtained from Figure 8 that S2's answer still wrong. When S2 was interviewed, S2 replied that he does not understand how to abstract the logarithm characteristic in the question. So, S2 has not been capable fulfil indicator abstraction in algebraic thinking skill.

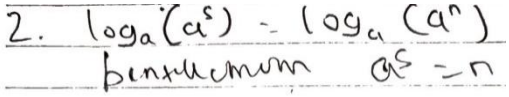

Figure 8. S2's Answer to Question Number 2

It can be seen in Figure 9 that S2 is able to determine the numerus for expression logarithm $\log_2 x = 3$ correctly. Then, S2 can also determine the numerus for expression logarithm when the basis changed becomes 4. So, indicator dynamic thinking in algebraic thinking skill was satisfied by S2.

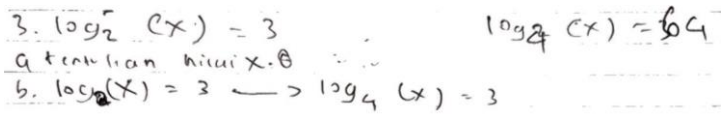

Figure 9. S2's Answer to Question Number 3

It is obtained from Figure 10 that S2 has not been capable fulfil indicator analytic thinking because the answer still wrong. When S2 was interviewed, S2 replied that he cannot determine the solution from logarithm equality, so that S2 cannot analyze whether the answer presented in the question is correct or still wrong.

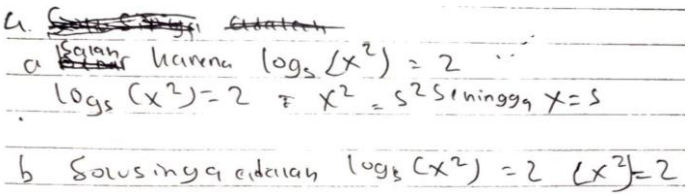

Figure 10. S2's Answer to Question Number 4

In question 5 and 6, S2 just write the question without answer. When S2 was interviewed, S2 replied that he cannot solve the logarithms equality presented in the questions number 5. So, can known that S2 has not been capable determine solution

from a logarithm equality. For question number 6, S2 has difficulty in solving the logarithm problem presented in the question number 6. In this case, S2 has not been capable fulfil indicator organizing and modeling in algebraic thinking skill. Yusrina & Masriyah (2019) state that when students do not fulfil indicator abstraction, then students also do not fulfil indicator analytic thinking and modeling.

In general, S2 satisfy the indicator generalization and dynamic thinking algebraic thinking skill.

3.3 Algebraic Thinking Skill of Student with the Lowest Score

It can be seen in Figure 11 that S3 is capable generalize logarithm characteristic for three variable based on logarithm characteristic presented in the question. So that, S3 satisfied indicator generalization in algebraic thinking skill.

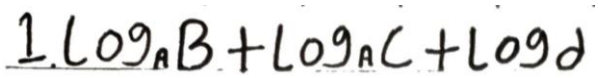

Figure 11. S3's Answer to Question Number 1

In question 2 to 6, there is not any questions were answered correctly by S3. When confirmed through an interview, S3 answered that he did not understand what was known and what was asked in the questions, so S3 could not solve the problems presented. Based on the result, S3 has not been able to satisfied the indicators abstraction, dynamic thinking, analytic thinking, organizing, and modeling in algebraic thinking skill.

In general, S3 just fulfil indicator generalization in algebraic thinking skill. This result is in line with the results presented by Aprildat & Hakim (2021) in their research, where students can fulfil indicator generalization in algebraic thinking skill. Badawi, Rochmad, & Agoestanto (2016) also stated that in general, students show the best abilities in generalization activities.

CONCLUSION

Based on the results, after the learning process with the discovery learning model, student with the highest score satisfy 5 indicators of algebraic thinking skill. Student with the highest score cannot fulfil indicator anaytic thinking because he did not realize the solution of square root in negative number. Students with score around the average just satisfy 2 indicators algebraic thinking skill, that is generalization and dynamic thinking because he has difficulty in determine the solution of logarithm equality and also has difficulty in solving the logarithm problem. Student with the lowest score just satisfy indicator generalization in algebraic thinking skill because he cannot understand what is known and asked in the question, so he cannot solve the problem presented.

The learning process affects students' algebraic thinking skill. The use of appropriate learning models can increase the effectiveness of the learning process to support the development of students' algebraic thinking skill. In this study, the discovery learning model has been used.

Teachers are advised to be able to apply the right learning model to develop students' algebraic thinking skill. In addition, further research is needed regarding students' algebraic thinking skill with other learning models so that the description of students' algebraic thinking skills can be more complete. It is also important that when conducting tests, the processing time needs to be considered so that students can answer each question optimally.

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