ANALYSIS OF A FOUR-BAR MECHANISM USING THE COMPLEX NUMBER METHOD WITH MATLAB PROGRAMMING

[1] **Ayu Wulandari**,*[2]**Muchlisinalahuddin**,[3]**Riza Muharni**
[4]**Yuni Vadila**,[5]**Reyhan Stevano**

[1,2,3,5]Mechanical Engineering, Faculty Engineering, Universitas Muhammadiyah Sumatera Barat
Padang, Indonesia
[4]Mechanical Engineering, Faculty Engineering, Universitas Negeri Padang
Padang, Indonesia

Author's email:
[1]ayu12.aw@gmail.com;[2] muchlisinalahuddin.umsumbar@gmail.com;[3] rizamuharni12@gmail.com*
[4] yunifadilabkt@gmail.com; *[5] reyhanstevano55@gmail.com*

**Corresponding author: muchlisinalahuddin.umsumbar@gmail.com*

Abstract. *This study aims to analyze the kinematic characteristics of a four-bar linkage mechanism applied to an automotive wiper system using the complex number method implemented through MATLAB programming. The four-bar mechanism is modeled based on actual geometric parameters obtained from AutoCAD drawings, consisting of a crank, a connecting rod, and a rocker with two fixed pivot points. The complex number method is used to formulate the loop-closure equations so that the relationships of position, angular velocity, angular acceleration, as well as linear velocity and acceleration at key points of the mechanism can be systematically analyzed. Numerical simulations are performed by applying a constant angular velocity to the crank. The analysis results show that the connecting rod rotates counterclockwise, while the rocker rotates clockwise, in accordance with the motion characteristics of the wiper mechanism. The magnitudes of velocity and acceleration in each link and at critical points are significantly influenced by the link length configuration and the input angular velocity. Validation of the calculation results using a parallel numerical method shows a very small relative difference, thereby confirming the reliability of the developed model. The MATLAB based complex number method is proven to be effective and accurate for the kinematic analysis of four-bar mechanisms and can serve as a reference for the design and development of wiper mechanisms and similar planar mechanisms.*

Keywords: *Car Wiper Mechanism; Complex Number Method; Four-Bar Mechanism; Kinematics; MATLAB.*

1. INTRODUCTION

Mechanisms are a fundamental part of mechanical engineering, serving to transform motion and force according to the requirements of a system. One of the simplest mechanisms with wide-ranging applications is the four-bar mechanism. The four-bar mechanism is one of the most commonly used mechanisms for converting one type of motion into another. This mechanism is widely applied in various mechanical systems, such as vehicle wiper mechanisms, crank–connecting rod systems, and automatic machinery, due to its simple construction and its ability to generate a wide variety of motions (R. L. Norton, 2011).

In the study of mechanics, mechanism analysis is divided into two main parts, namely kinematic analysis and dynamic analysis. Through kinematic analysis, the four-bar mechanism aims to determine the relationships of position, velocity, and acceleration among the links without considering the forces causing the motion. Conventionally, kinematic analysis can be performed using graphical methods and trigonometric methods. However, these methods tend to be less efficient when applied to numerical analysis and computational simulation. An understanding of kinematic aspects is very important because it serves as the basis for designing efficient mechanical systems before conducting force or dynamic analysis (R. L. (Robert L. . Norton, 2018; Uicker, J. J; Pennock, G. R; Shigley, 2010).

One effective mathematical approach for the kinematic analysis of planar mechanisms is the complex number method. This method allows the representation of link position vectors in complex exponential form so that the loop-closure equations can be formulated concisely and systematically. With this approach, the differentiation process to obtain velocity and acceleration becomes simpler. The complex number method is capable of improving the accuracy of planar mechanism design, particularly in generating mixed motion and specific functions (function generation) more precisely (R. L. Norton, 2011; Yogi Saputra et al., 2022).

Along with the development of computational technology, MATLAB (Matrix Laboratory) software is widely used in mechanism analysis due to its capability in handling numerical calculations, complex numbers, and motion visualization (Arman et al., 2017). Therefore, the use of the complex number method implemented through MATLAB programming provides a comprehensive, accurate, and structured approach to analyzing the kinematics of four-bar mechanisms (He & Guest, 2024; Pratap, 2021). MATLAB also enables real-time visualization of mechanism motion, which is very helpful in the process of analysis and model validation. (He & Guest, 2024; Pratap, 2021). MATLAB also enables real-time visualization of mechanism motion, which is very helpful in the process of analysis and model validation (MathWork, 2023).

Based on these considerations, this study aims to apply the complex number method in the kinematic analysis of a four-bar mechanism for automobile wiper applications. The position, velocity, and acceleration parameters are numerically analyzed using MATLAB programming in order to comprehensively describe the motion characteristics of the mechanism.

2. LITERATURE REVIEW

2.1 Four-Bar Mechanism

The four-bar mechanism (four-bar linkage) is the simplest planar mechanism and is most frequently used in many mechanical devices to achieve specific processes or motions (Myszka, 2012). The four-bar mechanism consists of four rigid links connected by four revolute joints forming a closed loop. These four links include the fixed link (ground link), the driving link (input link), the connecting link (coupler link), and the output link. The combination of link lengths and joint configurations determines the type and motion characteristics of the mechanism (R. L. Norton, 2011; Uicker, J. J; Pennock, G. R; Shigley, 2010).

The four-bar mechanism is widely used because of its ability to convert rotational motion into oscillatory motion or other complex motions, depending on its configuration and geometric parameters (Nazaruddin & Fatra, 2023). The motion configuration of the mechanism can be determined based on the relationship of link lengths known as Grashof's law. If the sum of the lengths of the longest and shortest links is less than the sum of the other two links, the mechanism is classified as a Grashof mechanism, which allows full rotation of the shortest link. This criterion facilitates the determination of the type of motion that can be produced by the mechanism, such as double-crank, crank-rocker, or double-rocker motion. (Kimbrell, 2006; R. L. Norton, 2011).

2.2 Planar Kinematics of the Four-Bar Mechanism

Planar kinematics is a branch of mechanics that studies the motion of bodies in a two-dimensional plane without considering the forces causing the motion. In the analysis of a four-bar mechanism, planar kinematics is used to determine the relationships of position, velocity, and acceleration among the links as the mechanism moves. (Hibbeler, 2016).

In planar mechanisms, all motion occurs in a single plane, so each member of the mechanism undergoes changes in position and orientation in two dimensions only. This concept is important as the basis for analyzing the motion of four-bar mechanisms (Hibbeler, 2016; R. L. Norton, 2011). In a four-bar mechanism, kinematics usually involves several main components as follows (R. L. Norton, 2011):

1. Angular relationships between links $(\theta_1, \theta_2, \theta_3, \theta_4)$;
2. Link lengths (r_1, r_2, r_3, r_4);
3. Loop closure equation;
4. Time derivatives for velocity and acceleration.

2.3 Complex Number Method in Mechanism Analysis

The complex number method is one of the methods used in numerical analysis for solving differential equations (Tanti et al., 2014). In the complex number method, the position vector of a link is represented as a complex number, which is mathematically expressed as:

$$z = x + iy \qquad (1)$$

Where:
$x =$ riil
$y =$ imajiner
$i = \sqrt{-1}$

Or in exponential form :

$$r_i = r_i e^{i\theta_i} \qquad (2)$$

Where :
r_i = length of the i-th link
θ_i = angle of the link with respect to the horizontal axis
i = imajinary unit $(j^2 = -1)$

The representation of link vectors in complex form is based on Euler's identity, which states that:

$$e^{i\theta} = \cos\theta + i\sin\theta \qquad (3)$$

Equation 3 shows that the multiplication of a complex number by a vector is equivalent to a rotation of the vector in the complex plane by an angle θ. Therefore, a link vector with length (r) and orientation angle θ with respect to the reference axis can be written as:

$$r_i = r_i(\cos\theta_i + i\sin\theta_i) \qquad (4)$$

This method is commonly used in the analysis of position, velocity, and acceleration because its computations are simpler and can be implemented in MATLAB. (Tanti et al., 2014; Yogi Saputra et al., 2022).

3. RESEARCH METHODS

This type of research is a quantitative descriptive study that focuses on the analysis of a four-bar mechanism using a mathematical approach. The method used is a kinematic analysis method based on mathematical modeling, specifically employing the complex number method. The research begins with problem identification to determine the data issues to be analyzed using the complex number method and MATLAB programming. This is followed by a literature study to collect supporting theories from books, journals, and articles.

Next, the formulation of kinematic equations and the determination of four-bar mechanism parameters, including the length of each link and the position of the fixed joints, are carried out. The parameters are determined by considering Grashof's law. The next stage is the formulation of the kinematic equations of the four-bar mechanism using the complex number method in the form of loop-closure equations. Numerical simulations are then performed using MATLAB by varying the input angle of the driving link. The calculated angles of the coupler link and the output link are used to visualize the mechanism in two-dimensional form. The final stage involves analyzing the simulation results to observe the motion characteristics of the four-bar mechanism.

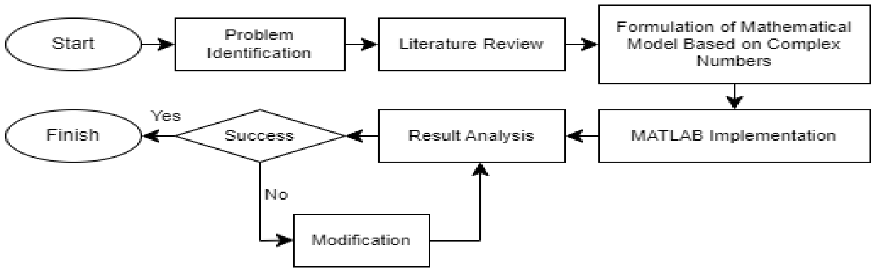

Figure 1. Research Flow Diagram

4. RESULTS AND DISCUSSION

The results of the kinematic analysis of the four-bar mechanism were obtained using the complex number method and MATLAB programming. The analysis was conducted to determine the relationships of position, velocity, and acceleration of the mechanism based on geometric parameters obtained from the actual model. The numerical calculation results were then analyzed to interpret the motion characteristics of the mechanism.

4.1 Geometric Model of the Mechanism

*B*ased on the field drive model, the analyzed mechanism is a four-bar linkage that is modeled and illustrated using AutoCAD software, as shown in Figure 2. The fixed points in this mechanism are:

1. O_2 and O_4 as fixed pivots
2. The distance between the two fixed pivots is O_2O_4 = 465 mm

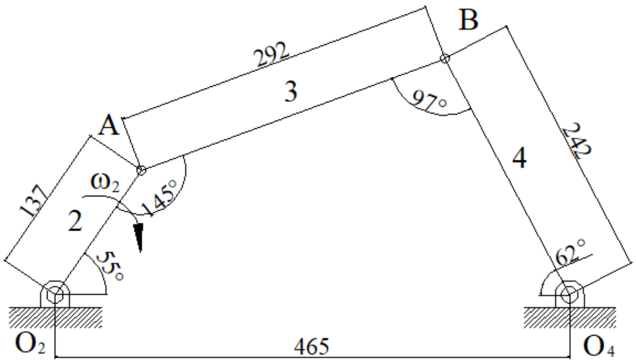

Figure 2. Four-Bar Mechanism

4.1 Mechanism Specifications and Parameters

The specifications of the lengths of each link in the four-bar mechanism are presented in **Table 1**.

Table 1. Specifications of the Four-Bar Mechanism

No	Link	Notation	Length (mm)
1	Crank	2 (O_2A)	137
2	Coupler	3 (AB)	292
3	Rocker	4 (BO_4)	242

The position angles of each link were obtained directly from measurements on the AutoCAD drawing (during analysis), namely:

Angle of link 2 with respect to the horizontal:

$\theta_2 = 55^0$

Angle between links 2 and 3:

$\theta_{23} = 145^0$

$\theta_3 = 55^0 - 145^0$

$\theta_3 = -90^0$

Angle of link 4 with respect to the horizontal:

$\theta_4 = 62^0$

These angles are used as initial data in the kinematic analysis of the mechanism.

4.2 Input Angular Velocity Data

Based on the mechanism model, link 2 (crank) rotates clockwise with an angular speed of $\omega_2 = 1200\ rpm$. It can then be calculated by converting it into rad/s using the following equation:

$$\omega_2 = \frac{2\pi(1200)}{60} = 125,66 \text{ rad/s}$$

Since the rotation is clockwise, the angular velocity is considered negative:

$$\omega_2 = -\frac{2\pi(1200)}{60} = -125,66 \text{ rad/s}$$

4.3 Position Analysis (Complex Number Method)

Position analysis is carried out using the complex number method by forming the loop vector equation of the four-bar mechanism as follows:

$$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = 0$$

Each position vector is expressed as a complex number using the Euler exponential function $\left(re^{i\theta}\right)$. The equation in Euler form becomes :

$$\mathrm{r}_2 e^{i\theta_2} + \mathrm{r}_3 e^{i\theta_3} = \mathrm{r}_1 + \mathrm{r}_4 e^{j\theta_4}$$

$$137e^{i\theta_2} + 292e^{i\theta_3} = 465 + 242e^{i\theta_4}$$

This equation is satisfied by the mechanism geometry obtained from the AutoCAD drawing.

4.4 Velocity Analysis (First Derivative)

Velocity analysis is obtained by taking the first derivative with respect to time of the complex position equation, resulting in:

$$\mathrm{i}\omega_2 r_2 \mathrm{e}^{\mathrm{i}\theta_2} + \mathrm{i}\omega_3 r_3 \mathrm{e}^{\mathrm{i}\theta_3} - \mathrm{i}\omega_4 r_4 \mathrm{e}^{\mathrm{i}\theta_4} = 0$$

By separating the real and imaginary components, two simultaneous equations are obtained ω_3 and ω_4, which are used to determine and. For numerical substitution, use:

$$\theta_2 = 55^0$$

$$\theta_3 = -90^0$$

$$\theta_4 = 62^0$$

After separating the components and solving the system of equations, it is obtained that:

$\omega_3 = 52,4$ rad/s (rotates counterclockwise)

$\omega_4 = 31,8$ rad/s (rotates clockwise)

Linear Velocity of Key Points

The linear velocity of point A is calculated using the following relation:

$$v_A = \omega_2 r_2 = 125,66(137)$$

$$v_A = 17215.42 \text{ mm/s}$$

The linear velocity of point B is obtained from the velocity polygon as well as from the complex number analysis:

$$v_B = \omega_4 r_4 = 31,8(242)$$

$$v_B = 7695,6 \text{ mm/s}$$

4.5 Acceleration Analysis (Second Derivative)

Since ω_2 is constant, then $\alpha_2 = 0$

The second derivative with respect to time of the complex position equation results in the absolute acceleration equations of the mechanism as follows:

$$-\omega_2^2 r_2 e^{i\theta_2} + \alpha_3 r_3 e^{i\theta_3} - \omega_3^2 r_3 e^{i\theta_3} + \alpha_4 r_4 e^{i\theta_4} + \omega_4^2 r_4 e^{i\theta_4} = 0$$

The equation is then separated into real and imaginary components to form two simultaneous equations, which are used to determine α_3 and α_4 as follows.

A. Acceleration Loop Using Complex Numbers

For the four-bar mechanism:

$$\vec{r_2} + \vec{r_3} - \vec{r_4} - \vec{r_1} = 0$$

The second derivative with respect to time, where $\alpha_2 = 0$. This is the absolute acceleration equation in complex form.

$$-\omega_2^2 r_2 e^{i\theta_2} + \alpha_3 r_3 e^{i\theta_3} - \omega_3^2 r_3 e^{i\theta_3} + \alpha_4 r_4 e^{i\theta_4} + \omega_4^2 r_4 e^{i\theta_4} = 0$$

B. Component Form (Euler)

The component form is given by the following equations:

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Based on the previous data, the geometric values are:

Table 2. Geometric Values

Angle	cos	Sin
550	0,574	0,819
-900	0	−1
620	0,469	0,883

Since the right-hand = 0, the real and imaginary components are separated as follows:

a) Real component

$$-\omega_2^2 r_2 \cos\theta_2 - \omega_3^2 r_3 \cos\theta_3 + \omega_4^2 r_4 \cos\theta_4 - \alpha_3 r_3 \sin\theta_3 + \alpha_4 r_4 \sin\theta_4 = 0$$

Then substitute :

$$-(125,66)^2(137)(0,574) - (52,4)^2(292)(0) + (31,8)^2(242)(0,469)$$
$$-\alpha_3(292)(-1) + \alpha_4(242)(0,883) = 0$$

Calculate the constant:

$$-1241728,27-0+114773,72-292\alpha_3+213,686\alpha_4=0$$

$$-292\alpha_3+213,686\alpha_4=1126954,55$$

b) Imaginary component

$$-\omega_2^2 r_2\sin\theta_2-\omega_3^2 r_3\sin\theta_3+\omega_4^2 r_4\sin\theta_4+\alpha_3 r_3\cos\theta_3-\alpha_4 r_4\cos\theta_4=0$$

Then substitute:

$$-(125,66)^2(137)(0,819)-(52,4)^2(292)(-1)+(31,8)^2(242)(0,883)$$

$$+292\alpha_3(0)-242\alpha_4(0,469)=0$$

Calculate the constant:

$$-1771734,24+801761,92+216087,83-113,498\alpha_4=0$$

$$-113,498\alpha_4=753884,49$$

C. Solution of the Imaginary and Real Equation System

From the imaginary equation, it is obtained:

$$\alpha_4=\frac{753884,49}{-113,498}rad/s^2\approx-6642\ rad/s^2$$

Substitute real component :

$$-292\alpha_3+213,686\alpha_4=1126954,55$$

$$-292\alpha_3+213,686(-6642,27)=1126954,55$$

$$-292\alpha_3-1419360,10=1126954,55$$

$$-292\alpha_3=1126954,55+1419360,10$$

$$\alpha_3=\frac{2546314,65}{-292}\approx-8720\ \text{rad/s}^2$$

D. Data Obtained from the Calculations

Link lengths

$r_2=137\ mm$

$r_3=292\ mm$

$r_4=242\ mm$

Angular Velocity

$\omega_2=125,66$ rad/s (CW)

$\omega_3=52,4$ rad/s (CCW)

$\omega_4=31,8$ rad/s (CW)

Angular Acceleration

$\alpha_2=0$

$\alpha_3=-8720\ rad/s^2$

$\alpha_4=-6642\ rad/s^2$

Linear Velocity

$$v_A = 17215,42\ mm/s$$
$$v_B = 7695,6\ mm/s$$

4.6 Calculating Linear Acceleration

A. Linear Acceleration Formulas

a) Centripetal (Normal) Acceleration

$$a_n = r\omega^2$$

b) Tangensial Acceleration

$$a_t = r\alpha$$

c) Total Acceleration

$$a = \sqrt{a_n^2 + a_t^2}$$

B. Linear Acceleration of Link 2 (Point A)

Since α_2=0 the linear acceleration of point A consists only of the normal (centripetal) acceleration, directed toward O_2

$$a_{A,n} = r_2\omega_2^2 = 137(125,66)^2$$
$$a_{A,n} = 2163289,68\ mm/s^2$$

C. Linear Acceleration of Link 4 (Point B)

The linear acceleration of point B from link 4 consists of two components (since B rotates about O_4), namely the normal component toward O_4 and the tangential component along the angular acceleration of link 4.

a) normal acceleration

$$a_{B,n} = r_4\omega_4^2 = 242(31,8)^2$$
$$a_{B,n} = 244720,08\ mm/s^2$$

b) tangensial acceleration

$$a_{B,t} = r_4\alpha_4 = 242(-6642)$$
$$a_{B,t} = -1607364\ mm/s^2$$

c) total acceleration

$$a_B = \sqrt{(a_{B,n})^2 + (a_{B,t})^2}$$
$$a_B = \sqrt{(244720,08)^2 + (-1607364)^2}$$
$$a_B \approx 1625886,51\ mm/s^2$$

D. Relative Linear Acceleration on Link 3 (AB)

The coupler link, the relative acceleration of point A with respect to point B consists of:

a) Relative Normal Acceleration

$$a_{BA,n} = r_3\omega_3^2 = 292(52,4)^2$$
$$a_{BA,n} = 801761,92\ mm/s^2$$

b) Relative Tangential Acceleration

$$a_{BA,t} = r_3\alpha_3 = 292(-8720)$$
$$a_{BA,t} = -2546240\ mm/s^2$$

c) Total Relative Acceleration of Link 3

$$a_{BA} = \sqrt{(a_{BA,n})^2 + (a_{BA,t})^2}$$

$$a_{BA} = \sqrt{(801761,92)^2 + (-2546240)^2}$$

$$a_{BA} \approx 2669486,90\ mm/s^2$$

Table 3. Summary of Linear Acceleration

No	Point/Link	Linier Acceleration
1	Point A	$2,16 \times 10^6$ mm/s²
2	Point B	$1,62 \times 10^6$ mm/s²
3	Relative AB	$2,66 \times 10^6$ mm/s²

4.7 Numerical Results and MATLAB Visualization of Four-Bar Mechanism Kinematics

The following graphs are generated using MATLAB based on the numerical solution of the complex number-based kinematic equations, illustrating the position, velocity, and acceleration characteristics of the four-bar mechanism over one complete crank cycle. This analysis provides a quantitative overview of the mechanism's behavior, which is important for the design and evaluation of the system's dynamic performance.

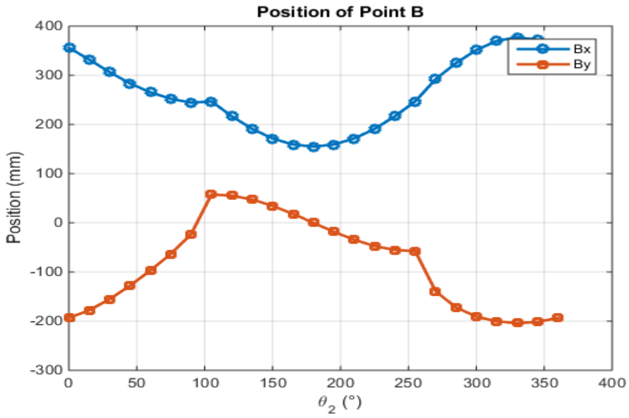

Figure 3. Position of Point B

Figure 3 shows the trajectory of point B during one full rotation of the crank (θ_2 = 0°–360°). The closed curve indicates a periodic motion characteristic of the four-bar wiper mechanism, where the rocker oscillates within a limited angular range.

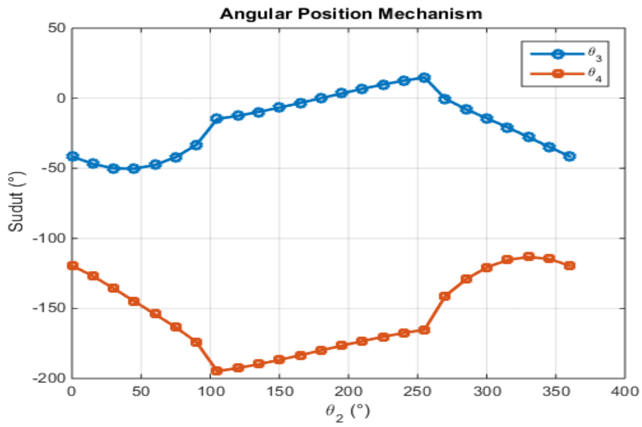

Figure 4. Angular Position Mechanism

This figure illustrates the angular position variations of the coupler and rocker links as a function of the crank angle. The results confirm that the coupler rotates continuously, while the rocker exhibits oscillatory motion, which is consistent with the operating principle of a wiper mechanism.

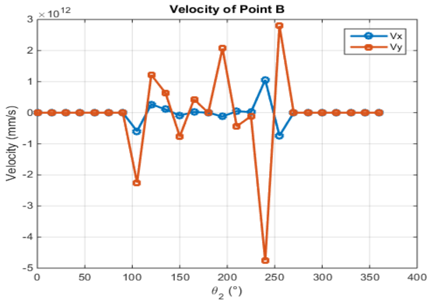

Figure 5. Angular Velocity of Point B

Figure 5 presents the angular velocity variation of point B throughout the motion cycle. The non-uniform velocity distribution indicates that the speed of the output link changes significantly depending on the crank position.

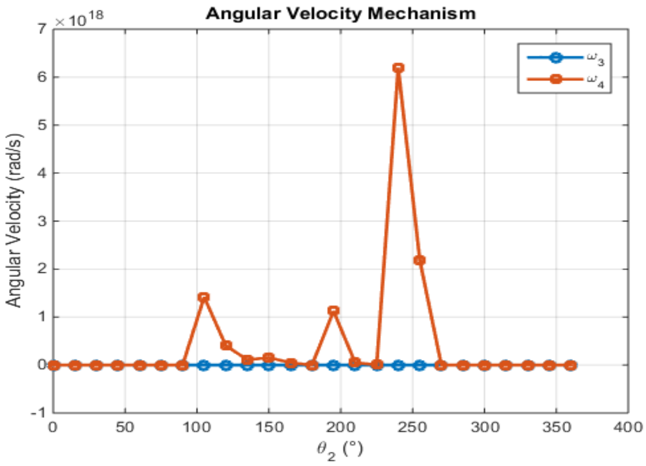

Figure 6. Angular Velocity Mechanism

The angular velocity curves of each link are shown in **Figure 6**. It can be observed that the coupler and rocker experience different angular velocity trends due to their geometric configuration and relative motion constraints.

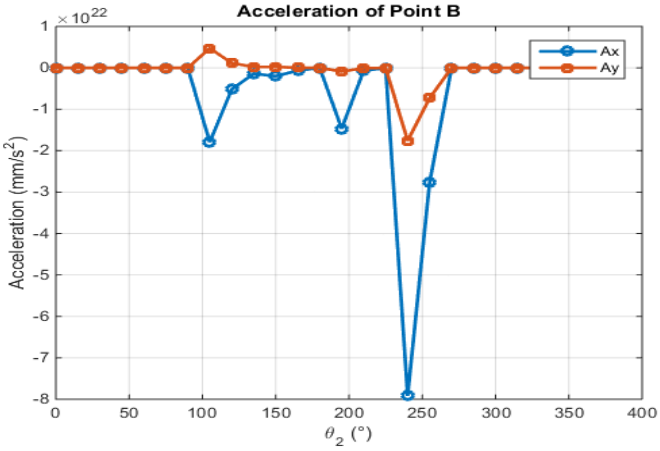

Figure 7. Acceleration of Point B

Figure 7 shows the acceleration of point B during the crank rotation. Peak acceleration values occur at specific crank angles, indicating positions where dynamic loads on the mechanism are potentially highest.

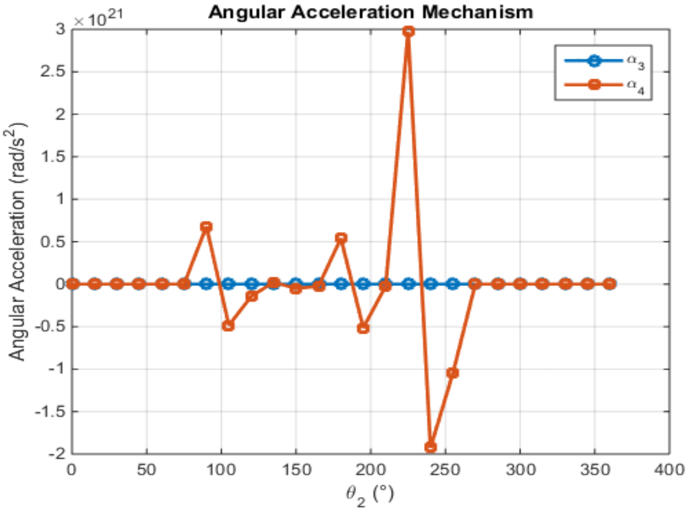

Figure 8. Angular Acceleration Mechanism

This figure illustrates the angular acceleration of the coupler and rocker links. The alternating positive and negative values indicate changes in rotational direction tendency, which are typical in oscillating mechanisms such as automotive wipers.

CONCLUSION

Based on the results of the kinematic analysis of the four-bar mechanism applied to an automobile wiper, the complex number method implemented through MATLAB programming is proven to be capable of systematically and accurately determining the relationships of position, velocity, and acceleration of the mechanism. A mechanism with a fixed pivot distance O_2O_4 of 465 mm and link lengths of 137 mm for the crank, 292 mm for the coupler, and 242 mm for the rocker exhibits motion characteristics consistent with a wiper mechanism. With an input angular velocity of the crank of 1200 rpm (CW), it is obtained that the coupler rotates counterclockwise (CCW) with an angular velocity of 52.4 rad/s, while the rocker rotates clockwise (CW) with an angular velocity of 31.8 rad/s.

The acceleration analysis results show that the angular accelerations of the coupler and the rocker are −8720 rad/s² and −6642 rad/s², respectively. The linear velocities at points A and B reach 17215,42 mm/s and 7695,6 mm/s, respectively, while the maximum linear accelerations occur at point A of $2,16 \times 10^6$ mm/s² and at point B of $1,62 \times 10^6$ mm/s². The MATLAB numerical calculation results are validated through parallel calculations using Microsoft Excel with a relative difference of less than 1%, thereby confirming the reliability of the developed kinematic model. Thus, the MATLAB-based complex number method can be used as a practical and accurate approach for the analysis and design of four-bar mechanisms in wiper systems and similar planar mechanisms.

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